Online Coding Event

Amalgam 2018

February 10, 2018

Part A

(Max Points : 40)

The Quantum Harmonic Oscillator is a problem where in an arbitrary potential can be approximated as a harmonic potential (quantum analog of a mass attached to a spring) in the neighbourhood of a stable equilibrium point. This is one of the handful of problems/potentials for which the time independent Schrödinger equation can solved analytically. Here, the 1-D harmonic potential is of the form $(\frac{kx^2}{2})$. The time independent Schrödinger equation in 1-D can be written as

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi, \tag{1}$$

Hence the problem can in essence be formulated as an eigenvalue problem of the form

$$\hat{H}\left|\psi\right\rangle = E\left|\psi\right\rangle,$$

Where, $\hat{H}$ is the hamiltonian operator.

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x), \tag{2}$$

E the eigen value is the time-independent energy level of the state $\left|\psi\right\rangle$. $\left|\psi\right\rangle$ by all means can be equated to the wavefunction ψ. This renders a second order differential equation in ψ. The solution of the above equation gives the wavefunction at different energy levels.

Solve the above equation to find the wavefunctions and corresponding energies for a hypothetical system with potential given by $V(x) = (\frac{kx^4}{2})$ and plot the wavefunction for different energy states.

The wavefunction is intricately related to the probability density of finding a particle at a particular position (since ψ is only a function of position here!). The probability density summed over all space should be equal to one. Hence the wavefunction must disappear at the two extremities , i.e ψ must die down at a far off distance.

Take,

$$\hbar = 1.0,$$
$$k = 1.0,$$
$$m = 1.0,$$

Make appropriate assumptions, if needed, and clearly state them.